A Marxian Model of the U.S. Long Waves with Endogenous Growth of Labour Force♥

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Abstract. This paper defines a hypothetical Law of capital accumulation that includes a growth rate of supply of labour force as a non-linear function of capital intensity. The main state variables are the labour productivity, unit value of labour force, employment ratio, and capital-output ratio. An application of an extended Kalman filtering to the U.S. macroeconomic data 1969–2002 and computer simulation runs demonstrate that long wave has been a viable pattern of capital accumulation.

The characteristic of the inertia scenario is a strengthening of the secular tendency of the general profit rate to fall. This is not accepted by the U.S. state and business leadership. The terrorist attack of the September 11, 2001 has served as a new powerful catalyst for a mobilising policy that aims at a fast overcoming of the structural crisis and safeguarding the global dominance based on technological leadership.

The mobilising policy enables, probably, overcoming the structural crisis, accelerating productivity growth, raising the general profit rate, reducing unemployment ratio in 2001–2010. The main leverage is freezing real wage under the conditions of the present war. This paper touches briefly the question whether the USA tend to participate in wars late in long boom and beginning of structural crisis.

1. The Hypothetic Law (HL) of Capital Accumulation

The HL upgrades models developed in [8] and [9]. The advanced capital does not include variable capital since workers advance capitalists. The HL abstracts from capital of circulation. Natural capital is not taken into explicit account in this paper.

1.1 An Extensive Deterministic Form of the HL

A deterministic model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates indicated by a hat. It consists of the following equations:

$$P = K/s; \tag{1.1}$$

$$L = P/a; \tag{1.2}$$

$$u = w/a; \tag{1.3}$$

$$\hat{a} = m_1 + m_2(K\hat{/}L) + m_3\psi(\hat{v}), \tag{1.4}$$

$$\psi(\hat{v}) = sign(\hat{v})|\hat{v}|^j, \ m_1 > 0, \ 1 > m_2 > 0, \ m_3 > 0, \ 1 > j > 0;$$

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$$\hat{K / L} = n_1 + n_2 u + n_3 (v - v_c), \tag{1.5}$$

$n_2 > 0, n_3 > 0, \ 1 > v_c > 0;$

$$v = L/N; \tag{1.6}$$

$$n = p_1 e_1^{-M_1 |K/L - K_c/L_c|^{i1}} \text{ for } 0 < K/L < K_c/L_c, M_1 = 1; \tag{1.7a}$$

$$n = p_1 e_2^{-M_2 (K/L - K_c/L_c)^{i2}} \text{ for } K/L \geq K_c/L_c, \ M_2 = 1, \ p_1 > 0; \tag{1.7b}$$

$$\hat{w} = -g + rv + b(\hat{K / L}), \ g > 0, r > 0; \tag{1.8}$$

$$P = Q + \dot{K} = wL + (1 - k)M + \dot{K}; \tag{1.9}$$

$$\dot{K} = k[(1 - u)P], \ 0 < k < 1. \tag{1.10}$$

Equation (1.1) postulates a technical-economic relation between the advanced constant capital (K), net output (P) and capital–output ratio (s). Equation (1.2) relates labour productivity (a), net output (P) and labour input, or employment (L). Equation (1.3) describes the relative wage, or unit value of labour power (u), as a ratio of real wage (w) to labour productivity. Equation (1.4) is an extended technical progress function. It includes: the rate of change of capital intensity, K/L and direct scale effect, $m_3 \psi(\hat{v})$; $|x| \geq 0$ is an absolute value of x; $sign(x) = -1$ for $x < 0$, $sign(x) = 1$ for $x \geq 0$. The non-linear continuous function $\psi(\hat{v})$ is analytical except at a singular point $\hat{v} = 0$ where its positive first derivative becomes infinite.

Equation (1.6) outlines the rate of employment (v) as a result of the buying and selling of labour–power. In the equation (1.8), the rate of change of the real wage rate (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rate of change of capital intensity (K/L) additionally. The capital intensity (K/L) is a proxy for qualification.

Mechanisation (automation) manifests itself in a growing capital intensity. The rate of change of capital intensity (K/L) in the equation (1.5) is a function of the relative wage (u), difference between the real employment ratio (v) and some base magnitude (v_c) that is lower than quasi-stationary employment ratio (v_a) defined below. A high relative wage and high employment ratio promote mechanization (automation) that shapes the labour supply.

Before reaching a critical magnitude, mechanisation (automation) pushes new demographic groups (children, women, aged, immigrants from less developed countries) into a labouring population (as far as qualification really or potentially satisfies technological requirements) thus chiefly accelerating the growth of supply of labour force. Afterwards mechanisation (automation) becomes mainly a decelerating factor for the growth of supply of labour force because a substantial part of working-age population does not possess adequate qualification for being hired or self-employed. Accordingly, the equations (1.7a) and (1.7b) determine the growth rate of labour force (N) as a non-linear continuous function of capital intensity. The growth rate of labour force is monotonically increasing for $K/L \leq K_c/L_c$, reaching an absolute maximum $n_{\max} = p_1$ at the point $K/L = K_c/L_c$; this rate is monotonically decreasing for $K/L \geq K_c/L_c$.

In the equations (1.9) and (1.10), the net formation of constant capital is $\dot{K}$, Q sums net export, final private and public consumption, $M = (1 - u)P$ is a total profit in real terms.

1.2 An Intensive Deterministic Form of the HL

The deterministic model in an intensive form, derived from the equations (1.1) – (1.10), consists of four non-linear ordinary differential equations (1.11) – (1.14):

$$\dot{a} = (m_1 + m_2 (n_1 + n_2 u + n_3(v - v_c)) + m_3 \psi(\hat{v}))a, \quad (1.11)$$

$$\dot{s} = (-m_1 + (1 - m_2)(n_1 + n_2 u + n_3(v - v_c)) - m_3 \psi(\hat{v}))s, \quad (1.12)$$

$$\dot{v} = (k\frac{1-u}{s} - (n_1 + n_2 u + n_3(v - v_c)) - n(sa))v, \quad (1.13)$$

$$\dot{u} = (-g + rv - m_1 + (b - m_2)(n_1 + n_2 u + n_3(v - v_c)) - m_3 \psi(\hat{v}))u. \quad (1.14)$$

It has a quasi-stationary state

$E_a = (a_a, s_a, v_a, u_a)$, where (1.15)

$$a_a = a_0 e^{i(t-t_0)}, s_a = k\frac{1-u_a}{i}, v_a = \frac{g + (1-b)i}{r}, u_a = \frac{i - n_1 - n_3(v_a - v_c)}{n_2}, i = \frac{m_1}{1-m_2}.$$

A quasi-stationary growth rate of constant capital, net output, real wage, labour productivity and capital intensities is the same: $\hat{K}_a = \hat{P}_a = \hat{w} = \hat{a} = K\hat{/}L = i$. At this quasi-stationary state, the value of constant capital, employment and labour force are fixed, i.e., $K_a \hat{/} a_a = \hat{L}_a = n_a = 0$. The quasi-stationary general profit rate is $(1 - u_a)/s_a = i/k$.

This quasi-stationary state E_a is dynamically unstable because $\psi'(\hat{v}) = j|\hat{v}|^{j-1}$ goes to positive infinity at $\hat{v} \to 0$. This substantial singularity explains also why the growth rate of labour productivity changes stepwise at local extrema of the employment ratio. Abruptness of economic crises follows from this essential singularity too.

1.3 A Probabilistic Form of the HL

For taking into account measurement errors and an impact of factors neglected in the model assumptions, the deterministic model (1.11) – (1.14) has been transformed in a stochastic model. This makes implicit allowances for short-term and middle-term economic fluctuations by specification of the random components. The latter model includes state equations and measurement equations for discrete moments of time

$$\mathbf{x}(n) = \mathbf{f}[\mathbf{x}(n-1)] + \mathbf{w}(n), \quad (1.16)$$

$$\mathbf{z}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n), \quad (1.17)$$

where $n = 1, 2, \ldots, N$ is an index of data samples, $\mathbf{x}(0)$ – a vector of an initial state of the system, $\mathbf{w}(n)$ – a vector of equations errors (driving noise), $\mathbf{v}(n)$ – a vector of measurement errors. The deterministic part $\mathbf{x}(n) = \mathbf{f}[\mathbf{x}(n-1)]$ corresponds to the system (1.11) – (1.14). The symbol **H** is for a rectangular matrix. The residuals are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain highly systematic, non-random components.

This paper applies a simplified version of an extended Kalman filtering (EKF), realised in the Vensim software developed by Ventana Systems, Inc. This soft-ware has enabled to estimate the unobservable components of the compact model (1.11) – (1.14) by a procedure of maximum likelihood.

2. An Inertia Scenario for the U.S. Economy Based on the HL

An application of the EKF to the U.S. macroeconomic data for the basal period 1969–2002 has identified unobservable components of the above stochastic model: $b \approx 0.540$, $e_1 \approx 2.5$, $e_2 \approx 100$, $i_1 \approx 0.2$, $i_2 \approx 0.4$, $g \approx 0.046$, $j \approx 0.342$, $k \approx 0.203$, $K_c / L_c \approx 0.098$, $m_1 \approx$

0.0067, $m_2 \approx 0.2357$, $m_3 \approx 0.015$, $n_1 \approx -0.246$, $n_2 \approx 0.347$, $n_3 \approx 0.6$, $p_1 \approx 0.03$, $r \approx 0.053$, $v_c \approx 0.925$, $i \approx 0.009$. The simulation, started at the magnitudes of the phase variables observed in 1969 ($a_0 \approx 0.0422$, $s_0 \approx 1.826$, $v_0 \approx 0.965$, $u_0 \approx 0.710$), has calculated the most probable (still sub-optimal) magnitudes of these four and other variables in the subsequent years.

The main variables have the following units of measurement: a [millions of chained 1996 dollars per worker per year], u, v [dimensionless], s [years]. Calculations of u and s are done with the nominators and denominators measured in current prices. The employment ratio v is for the civil labour force (without accounting hidden unemployment). Private and governmental produced non-residential fixed assets present the constant capital.

2.1 A Historical Fit of the HL in the Basal Period 1969–2002

The HL has passed behaviour reproduction tests. In particular, estimating its historical fit (Table 1), the Theil inequality statistics have been used [10].

Table 1. Decomposition of errors of the retrospective forecast for 1969–2002

Variable	RMSPE (%)	U^M	U^S	U^C
a	0.81	0.049	0.076	0.875
s	3.31	0.002	0.315	0.683
v	0.92	0.000	0.080	0.920
u	1.56	0.088	0.000	0.912
$(1 - u)/s$	4.36	0.076	0.087	0.838

The rather small root-mean-square percent errors (RMSPE) and prevailing non-systematic errors of incomplete co-variation (U^C) prove that this probabilistic model tracks the major variables observed in the basal period agreeably. Fig. 1 and Fig. 2 support this conclusion by demonstrating a certain likeness of simulated and realised trajectories.

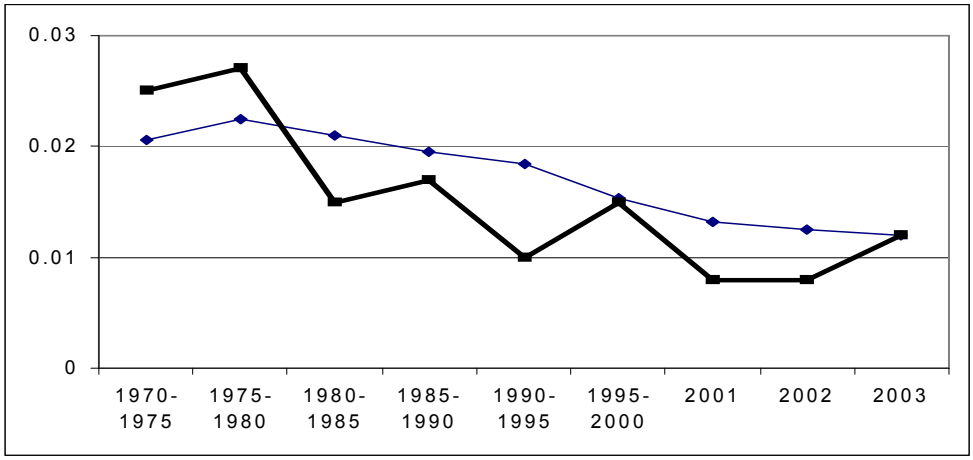

Fig. 1. The realised (solid broken line) [3: Table V.B2)] and simulated (thin one) growth rates of labour force (n) in the USA, 1970–2003

A long wave has been a viable pattern of the U.S. capital accumulation in the basal period with local maximum (minimum) of the employment ratio, v, in 2001 (1982) and local

maximum (minimum) of the general profit rate, $(1 - u_a)/s$, in 1998 (1982). The maximal magnitudes of the both variables are lower than their magnitudes in 1969. Moreover, the previous local maximum of the profit rate (higher than that in 1969) was observed in 1965–1966 before the basal period [9: 90–96]. The uncovered tendency of the profit rate to fall is unfavourable for the employment ratio in the long-term. A shortage of labour supply is detrimental for capital accumulation.

2.2 A Long-term Extrapolation of the Tendency of General Profit Rate to Fall

An extrapolation of the retrospective forecast, based on the deterministic model (1.1) – (1.10) with the parameters values given above, is called the inertia scenario.

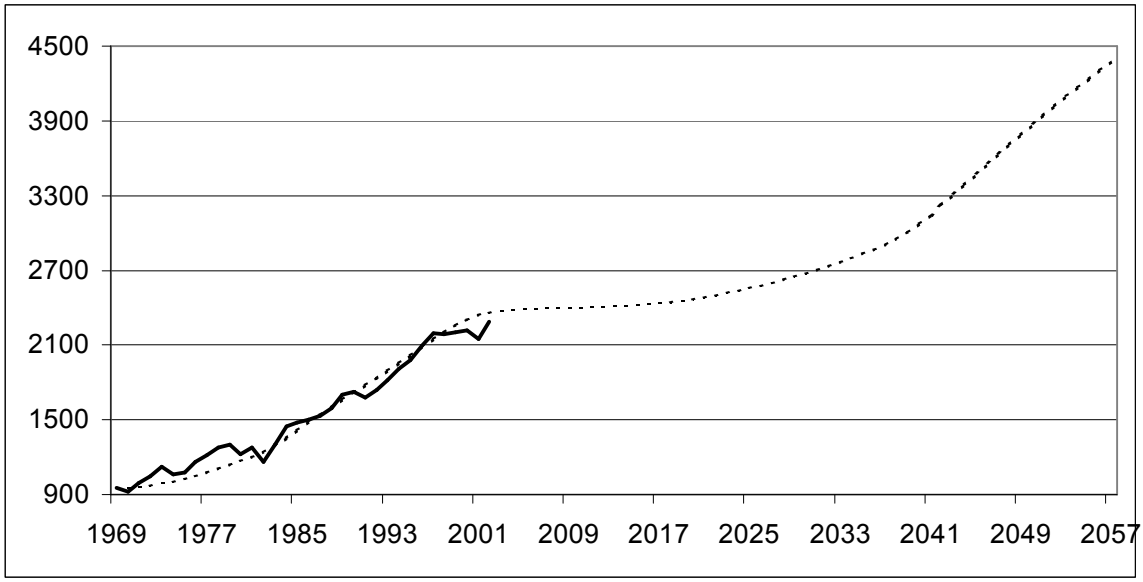

Fig. 2 The profit M (milliards 1996 dollar a year): realised (solid curve), 1969–2002, and simulated (dotted curve) in the inertia scenario, 1969–2057

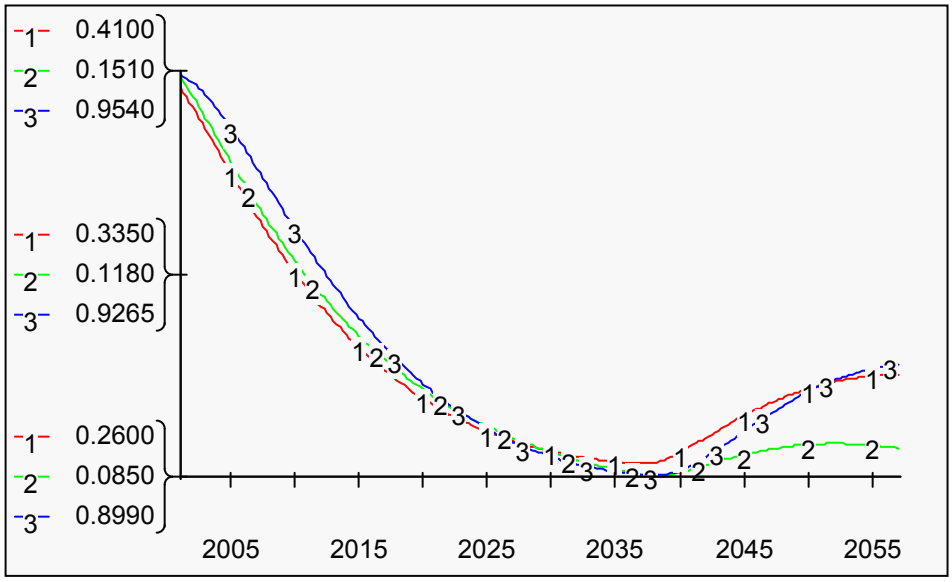

Fig. 3 The rate of surplus value $(1 - u)/u$ (1), rate of profit $(1 - u)/s$ (2) and employment ratio v (3), 2001–2057, in the inertia scenario

The tendency of the employment ratio, rate of profit and rate of surplus value to fall during the first quasi-cycle of the 21st century lasts until the end of 2030-s mainly because the growth rate of the real wage exceeds the growth rate of labour productivity. Only when the latter surpasses the former the long wave starts to move upwards.

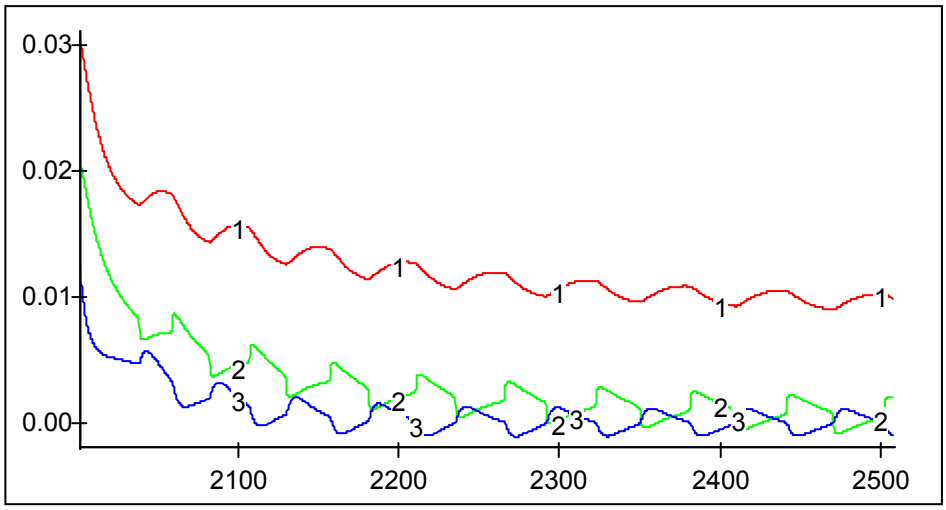

Fig. 4 The growth rates of the constant capital $\hat{K}$ (1), of its labour value $K\hat{/}a$ (2) and of labour input $\hat{L}$ (3) in the inertia scenario, 2001–2507

Profit in real terms grows uninterruptedly in spite of the fall in the profit rate in 2001–2038 (Fig. 2 and Fig. 3). Still this variable almost comes to a standstill when the profit rate declines.

Computer simulations reveal that phase variables (s, v, u), gross profit rate, growth rates of labour productivity and real wage as well as some other variables fluctuate. The duration of fluctuations is 58–63 years. The periods of fluctuations are shorter at the beginning. For example, the first complete quasi-cycle of the employment ratio (v) in the 21st century encompasses 2001–2058. More than four hundred years later, this variable starts to oscillate about the quasi-stationary value (v_a) with a period 62–63 years.

The growth rate of the material substance of the constant capital (K) and growth rate of its labour value ($K\hat{/}a$) as well as growth rate of the labour input ($\hat{L}$) experience the long-term anharmonic fluctuations. These growth rates, together with the general profit rate, tend to decline at the transient to the quasi-stationary values (Fig. 4).

3. Explaining a Contemporary Development of the U.S. Economy by a Modified HL

3.1. A Dialectical Negation of the Inertia Scenario

The inertia scenario above may lead to a wrong fatalistic conclusion that the general profit rate has inevitably to decline uninterruptedly in 1999–2038 and that the total profit is to be nearly constant in 2000–2010. The official middle-term macroeconomic projection in January 2001, based on information as of November 2000, carried traits of this pessimistic vision: the full amount of corporate profits (before taxes) in the year 2010 deflated by CPI was projected only 4 per cent higher than that in 2000 [7: Table II-1]. The same official middle-term projection envisioned that the ratio of the full amount of nominal corporate

profits (before taxes) to wages and salaries would have to decline from 0.196 in 2000 to 0.153 in 2010 (ibid.). The analogous rate of surplus value declines (Fig. 3) during these years in the inertia scenario too.

U.S. capital and leading circles have rejected a policy of a passive adaptation to the long-term decline. They have been carrying out a mobilizing policy at least since the beginning of 2001. The terrorist attack of the September 11, 2001 has served as a new powerful catalyst for this policy that aims at a fast overcoming of the structural crisis and safeguarding the global dominance based on technological leadership.

Second time after the World War II, the USA have started or become involved in the wars in the former Yugoslavia, Afghanistan and Iraq (1999 – nowadays) late in the boom of the previous big cycle and during the unfinished structural crisis of the current one. Recall that the U.S. wars in Indochina extended over 1965–1973 when the similar transition from the boom of one big cycle to the crisis of the other took place.

The wars, aimed at strengthening geo-political positions, serve capital to restore profitability. In particular, it is much easier for captains of finance and industry to explain workers a necessity to fasten their belts during a war than in peaceful time. Rephrasing saying from the J. Stainbeck 'Grapes of Wrath', *got enough wars and profit will hit the ceiling*. Cheapening elements of constant capital, foreign trade and outsourcing belong to additional counteracting factors beyond the scope of the present analysis.

An official middle-term projection based on information available in November 2001 has aimed at substantially higher growth of corporate profits in real terms and considerably higher ratio of nominal corporate profits to wages and salaries than those in the previous official projection: in 2010 the first indicator would have to stand higher by 23.1 per cent than in 2000, while the second would be 0.169 in 2010 [4: Table 2-1].

Output per hour worked grew since the fourth quarter of 2000 up to the beginning of 2004 at an exceptional annual rate of more than 4 per cent per year [6: 46]. Yet workers' compensation has consistently lagged productivity growth over this period. Total labour compensation has experienced the slowest growth in any recovery since World War II [2]. As a result, the profit share (ratio of property and entrepreneurial income to GDP) has recently reached its previous peak of 1997 – 7 per cent above its average for 1981–2003 [1: 15, 24]. This is the fastest rate of profit growth in a recovery since World War II.

3.2 A Synthesis of the HL and Historical Contingency

The inertia scenario above and facts from the previous section contradict each other like thesis and antithesis. Synthesis necessitates breaking the closeness of the initial causal system by saving its essence and allowing for the mobilizing policy. A working assumption is that the rate of growth of real wage is not higher in the middle-term 2001–2010 than the quasi-stationary magnitude, defined by the equation (1.15): $\hat{w} \leq \hat{w}_i \leq i$. The deliberately chosen magnitude $\hat{w}_i = 0.007$ is plausible. The modified equation for the rate of growth of real wage takes the form

$$\hat{w} = \min[\hat{w}_i, -g + rv + b(\hat{K/L})], \; 0 \leq \hat{w}_i \leq i \approx 0.009, \; g \geq 0, \; r > 0. \qquad (1.8')$$

All other equations, the starting point for 2001, and parameters values remain the same.

Table 2 reports on results of the simulation run based on the modified model. The outcomes of the mobilizing policy do not contradict qualitatively the above latest data on growth and distribution. They are compared with outcomes of the inertia scenario. For capital, the mobilising scenario is superior to inertia scenario. In particular, the total profit in the mobilising scenario will be 42.9 per cent higher in 2010 than in 2001 (in the inertia sce-

nario only 2.6 per cent higher). A recent official projection expects the 68.9 per cent increment of corporate profits (before taxes) deflated by CPI in 2001–2010 [5: Table C-1].

Table 2. Increments (per cent) in the two scenarios of the U.S. economic development, 2001–2010 (2001 = 100)

Variable	Scenario	
	Inertia	Mobilising
Labour productivity (a)	8.8	12.2
Real wage (w)	14.4	6.5
Rate of surplus value ($(1 - u)/u$)	–16.9	18.5
Profit rate ($(1 - u)/s$)	–19.8	7.5
Employment (L)	7.9	13.2
Labour force (N)	10.2	10.4
Surplus value ($(1 - u)L$)	–5.7	27.4
Constant capital (K)	27.9	32.9
Value of constant capital (K/a)	17.5	18.5
Net output (P)	17.4	27.0
Profit (M)	2.6	42.9

According to the simulation run, the American economy will crash into the upper limit of full employment ($v \approx 0.978$) at the end of the projection period or even before 2010 in the mobilising scenario. This creates the necessary conditions for a new crisis.

Conclusion

The HL of capital accumulation and computer simulations based on the U.S. official statistics have put in a nutshell how under conditions of the current war American workers have got to take freezing real wage to restore profitability and secure higher employment in coming years. The mobilising policy facilitates the U.S. technological, economic and military power while the labour distributive share shrinks. Still with all its benefits, especially for capital, the mobilizing scenario contains seeds of its own negation since capital itself remains the real barrier of capitalist production.

The presented HL could be useful in controlling severity of structural crises and helpful for avoiding wars as profitability instruments.

References

[1] Bank for International Settlements (BIS). 2004. The 74th Annual Report. Basel: Bank for International Settlements.

[2] Bivens J. Snapshot for May 27, 2004 / Economic Policy Institute (http://www.epinet.org).

[3] Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds. 2004. The 2004 Annual Report. Washington, D.C.: U.S. GPO.

[4] Congress of the United States. Congressional Budget Office. 2001. Economic and Budget Outlook: Fiscal Years 2002–2012.

[5] Congress of the United States. Congressional Budget Office. 2004 (September). The Budget and Economic Outlook: An Update. A Report to the Senate and House Committees on the Budget.

[6] Economic Report of the President. 2004. Washington, D.C: U.S. GPO.

[7] Executive Office of the President. 2001 (January). FY 2002 Economic Outlook, Highlights from FY 1994 to FY 2001, FY 2002 Baseline Projections. Washington, D.C: U.S. GPO.

[8] Ryzhenkov A. 2000. Unfolding the Eco-wave: Why Renewal Is Pivotal. Chichester, New York a. o.: John Wiley & Sons, Inc.

[9] Ryzhenkov A. 2003. Models of Cyclical Growth. Novosibirsk: IEIE SB RAS (in Russian).
[10] Theil H. 1966. Applied Economic Forecasting. Amsterdam: North-Holland Publishing Company.